

Discontinuous Galerkin Methods

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Discontinuous Galerkin (DG) methods can be viewed as finite element methods allowing for discontinuities in the discrete spaces. Localizing test functions to single mesh elements and introducing numerical fluxes at mesh interfaces, they can also be viewed as Finite Volume methods in which the discrete solution is represented on each mesh element by a polynomial function and not only by a constant function. DG methods in various forms have existed for more than three decades. However, DG methods have experienced a vigorous development mainly over the last decade. The extensive interest in DG methods is essentially fuelled by the substantial flexibility they offer, e.g., in using non-matching grids or varying polynomial degrees. At the same time, basic conservation principles can be easily incorporated into the method.

This lecture will focus on the mathematical foundations of DG methods by examining first elliptic problems and then symmetric first-order PDE's systems known as Friedrichs' systems so as to provide a unifying viewpoint for the design and analysis of DG methods. Finally, some aspects related to unsteady problems will also be discussed.