# A posteriori error estimates for adaptive mesh refinement and error control

The lecture surveys some recent results in a posteriori estimation of the error in numerical approximation of model partial differential equations. The emphasis is put on the estimates which are optimal in the following sense:

1) deliver an upper bound on the error in the numerical solution which only uses the approximate solution and which can be fully, without the presence of any unknown quantities, evaluated (**guaranteed upper bound**);

2) give an expression for the estimated error locally, for example in each element of the computational mesh, and ensure that this estimate on the error represents a lower bound for the actual error, up to a generic constant (**local efficiency**);

3) ensure that the ratio of the estimated and actual error goes to one as the computational effort grows to infinity (**asymptotic exactness**);

4) guarantee the three previous properties independently of the parameters and their variation (**robustness**);

5) give estimators which can be evaluated locally (**negligible evaluation cost**).

Property 1) allows to give a certified error upper bound, 2) is crucial for the suitability of the estimates for adaptive mesh refinement, 3) and 4) ensure the optimality of the upper bound, and 5) guarantees that the evaluation cost will be much smaller than the cost required to obtain the approximate solution itself.

The estimates will be presented for different numerical methods (continuous and discontinuous Galerkin finite element methods, finite volume and finite difference methods, mixed finite element method). Linear second-order elliptic convection–diffusion–reaction problems will be treated first and an extension to parabolic and nonlinear problems will be mentioned later. The case of inexact solution of the associated linear systems will be also addressed. Numerical results will illustrate the theoretical developments.