An hybrid finite volume - finite element scheme for variable density incompressible Navier-Stokes equations

Caterina Calgaro[1,2], Emmanuel Creusé[1,2] & Thierry Goudon[1,2]

[1]Laboratoire Paul Painlevé, UMR CNRS 8524, Université des Sciences et Technologies de Lille, F-59655 Villeneuve d'Ascq Cedex, France.

[2]EPI Simpaf, INRIA Lille Nord Europe, Parc scientifique de la haute borne, avenue Halley, BP 70478, F-59658 Villeneuve d'Ascq Cedex, France.

Winter School : "New trends in scientific computing",
CIRM, February 9th-13th 2009.

This talk will be devoted to the numerical simulation of variable density incompressible flows, modeled by the Navier-Stokes system :

$$\partial_t \rho + \mathrm{div}_{\mathbf{x}}(\rho \mathbf{u}) = 0, \tag{1}$$

$$\partial_t(\rho \mathbf{u}) + \mathrm{Div}_{\mathbf{x}}(\rho \mathbf{u} \otimes \mathbf{u}) + \nabla_{\mathbf{x}} p - \mu \Delta_{\mathbf{x}} \mathbf{u} = \mathbf{f}, \tag{2}$$

$$\mathrm{div}_{\mathbf{x}} \mathbf{u} = 0. \tag{3}$$

This problem combines the difficulty of the transport equation of the density (1) with the other of guaranteeing the divergence free constraint (3) in the evolution of the velocity (2). The originality of this work relies in the fact we consider different numerical methods to solve the mass conservation and to evaluate the velocity and the pressure driven by the momentum equation and the constraint. More precisely, using a time-splitting, (1) is solved for a given velocity field by a Finite Volume scheme which is well adapted when dealing with a pure convection equation, and then the divergence free solution of (2)-(3) is computed by exploiting the advantages of Finite Elements methods. The key point relies on the compatibility of the two approaches, because of the divergence free constraint which has to be preserved between the two steps of the splitting. Consequently, a suitable footbridge has to be constructed in order to link the two methods. This hybrid finite volume/finite element scheme will be here described in details, and several numerical tests will be presented to underline its capabilities. In particular, the viscous Rayleigh-Taylor instability evolution which shows the behaviour of an heavy fluid into a lighter one evolving in a cavity subject to a gravity field (see figure 1 below).

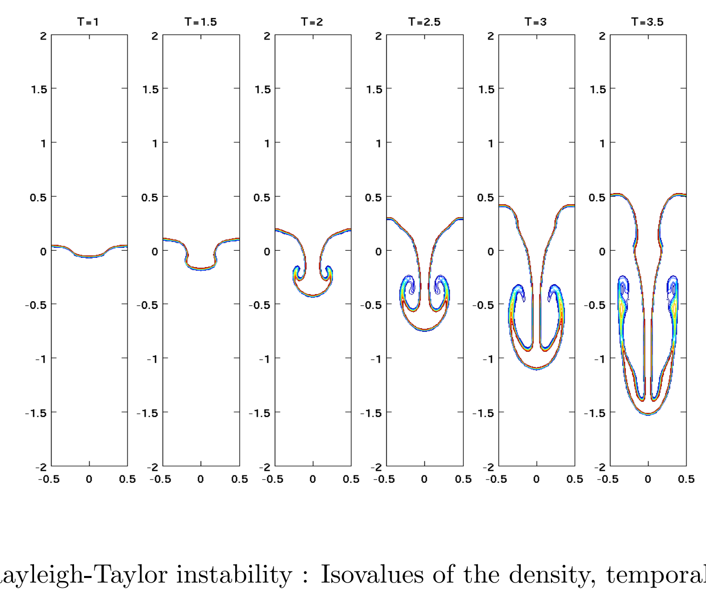

Figure 1: Rayleigh-Taylor instability : Isovalues of the density, temporal evolution.