Space-Time Expansion Discontinuous-Galerkin Method for Nonlinear Conservation Laws

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Discontinuous Galerkin schemes for hyperbolic conservative laws are now well-established since a long time. To improve the efficiency still new variants are developed, for example the so-called Space-Time Expansion Discontinuous-Galerkin Method (STE-DG) proposed by Lörcher et al. [1]. Time discretisation is here done by substituting time derivatives by space derivatives using the Cauchy-Kovalevskaya technique. This choice is not unique and we will show other examples. They all belong to a new class of Discontinuous Galerkin Methods.
Shu et al. presented error estimates for the Runge-Kutta DG-Method in [3]. As the main new contribution we present in [2] an error estimate for this new class of Discontinuous Galerkin Methods in 1D for smooth solutions. Under the CFL-condition $\tau \leq \gamma h^2$, $\gamma > 0$, we achieve an error estimate of $O(h^{k+\frac{1}{2}})$ for the $\mathbf{P}^k$ elements and a general monotone numerical fluxes. Here τ denotes the maximum time step and h denotes the maximum element lengths. For upwind numerical fluxes we obtain an error estimate of $O(h^{k+1})$.
Finally numerical results are shown to illustrate the theoretical findings.

References

[1] F. Lörcher, G. Gassner, C.-D. Munz: *A Discontinuous Galerkin Scheme Based on a Space-Time Expansion. I. Inviscid Compressible Flow in One Space Dimension.* In: *Journal of Scientific Computing* 32 (2007), pp. 175-199

[2] P. Engel: *Error Estimates to Smooth Solutions of Space-Time Expansion Discontinuous Galerkin Methods.* In preparation.

[3] Q. Zhang, C.-W. Shu.: *Error Estimates to Smooth Solutions of Runge-Kutta Discontinuous Galerkin Methods for Scalar Conservation Laws.* In: *SIAM J. Numer. Anal.* 42 (2004), pp. 641-666