A posteriori error estimates and stopping criteria for iterative solvers

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We consider the finite volume and the lowest-order mixed finite element discretizations of a second-order elliptic pure diffusion model problem. Our first goal is to derive guaranteed and fully computable a posteriori error estimates which take into account an inexact solution of the associated linear algebraic system. The algebraic error can be simply bounded using the algebraic residual vector. Much better results are, however, obtained using the complementary energy of an equilibrated Raviart–Thomas–Nédélec discrete vector field whose divergence is given by a proper weighting of the residual vector. Our second goal is to construct efficient stopping criteria for iterative solvers. We claim that the discretization error, implied by the given numerical method, and the algebraic one should be in balance, or, in other words, that it is enough to solve the linear algebraic system to the accuracy which guarantees that the algebraic part of the error does not contribute significantly to the overall error. Our estimates allow a reliable and cheap comparison of the discretization and algebraic errors. One can thus use them to stop the iterative algebraic solver at the desired accuracy level, without performing an excessive number of unnecessary additional iterations. Several numerical experiments illustrate our theoretical results.

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