Heterogeneous multiscale methods for scalar conservation laws with mixed local and nonlocal diffusion-dispersion terms

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It is well-known that the initial value problem for the scalar conservation law

$$u_t + f(u)_x = 0\,, \quad f \in \mathcal{C}^1(\mathbb{R}, \mathbb{R}) \tag{1}$$

with unknown $u : \mathbb{R} \times [0, T) \to \mathbb{R}$ can have multiple weak solutions. We are interested in solutions which are selected as the limit $u := \lim\limits_{\varepsilon \to 0} u^\varepsilon$ of the initial value problem for the regularized problem

$$u_t^\varepsilon + f(u^\varepsilon)_x = R^\varepsilon[u^\varepsilon]\,. \tag{2}$$

Here R^ε is a nonstandard viscosity-capillarity term that drives in particular nonclassical waves for (2) (cf. [1, 2, 3]). In this context (1) can be seen as a very simple model problem for the macroscopic dynamics of liquid-vapour (Euler equations) flow while (2) represents a microscale model (e.g. Navier-Stokes-Korteweg equations).

We present a numerical method to solve (1) with $u := \lim\limits_{\varepsilon \to 0} u^\varepsilon$ using a heterogeneous multiscale concept which is motivated by a algorithm for combustion fronts [4]. Close to locations of nonclassical waves the microscopic model (2) is used.

Numerical results that demonstrate the efficiency of the approach are presented. Moreover, we show that several methods to solve (1) directly on the basis of kinetic relations, can be elegantly rewritten in terms of the multiscale framework.

References

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