

Numerical Method for the Simulation of Free Surface Flows with Surface Tension

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Abstract :

The breaking of a wave, the splashing of a drop of water while impacting a solid surface are fascinating problems of fluid mechanics with free surfaces. Indeed, the liquid is bounded by a moving surface whose determination is part of the problem. This increases as much the mathematical difficulties of the solution. We are interested in some numerical aspects of problems with a singular forces of surface tension. The purpose of the talk is to present a numerical method for the simulation of two incompressible navier-stokes fluids separated by a free interface where the surface tension effects are imposed as a normal force. We use an eulerian approach and we localise the free surface with a "Level Set" method. The difficulties of this problem are the conservation of the volume of the domain bounded by the free surface, the computation of the curvature and the optimisation of numerical computations. The mathematical model has three problems : the advection problem, the navier stokes problem and the redistancing problem. the last one computed a new signed distance function from the original level set function having the same zero level set.

In the first part, we will deal with the transport and the redistancing problems, the purpose is to keep the conservation of the volume. A first approach is proposed by [1] and it was implemented by a finite differences method. We propose a generalisation of this method by finite elements and we add a second global constraint in order to make the advection step more accurate and efficient. Examples and verifications of the method are given by comparing simulations to analytical results for two classical tests : the Zalesac test and the Leveque test in the two dimensional case. Notably, we obtain a better rate of convergence for the L^1 error and we ameliorate the volume conservation compared with the classical method, which confirms the correct calculation of the advection and the redistancing steps.

In the second part, a method for the computation of surface tension effects is proposed here on an unstructured finite element mesh. The aim of this part is to adapt and to extend the method proposed in [2]. The scheme has been tested on different standard test cases of mean curvature motion with and without volume conservation.

In the last part, we add the fluid in the model by the incompressible navier stokes system and we show numerical simulations of droplets in the 2D and 3D axisymmetric cases. The different simulations are implemented with Rheolef[3].

Références :

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