Local adaptive refinement and multigrid method for multiphase flows simulations.

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We present a local adaptive finite element method combined with multigrid preconditioning, which exhibits the following attractive features: the same subdivision pattern is uniformly applied to all meshes; the generated grids are nonconforming but the multilevel finite element approximation spaces remain conforming by construction; a large independence is maintained between the particular PDEs, the adaptation procedure, the discretization scheme and the algebraic solver.

The principle of the adaptation procedure, called CHARMS [4, 1], is to refine/unrefine primarily basis functions instead of meshes. Given a coarse grid, possibly unstructured, the method relies on the existence of a conceptual hierarchy (i.e. not built in practice) of nested grids defined by uniform refinements. A parent/child relationship is then stated between basis functions belonging to Lagrange finite element spaces associated with two successive grids of this sequence. The elementary refinement (resp. unrefinement) is defined, without refering to grids, by the replacement of parents (resp. children) by their children (resp. parents).

At a given time t_n, the history of the time marching procedure leads to a multilevel approximation space V_J of functions belonging to different levels from the coarsest (level 1) to the finest (level J). Then a Galerkin method is used on the basis of V_J to approximate the unknowns at time t_n. The multilevel structure of V_J is further exploited to design efficient preconditioners of the arising linear system: the multigrid framework is applied on coarsened auxiliary embedded spaces $V_1 \subset \cdots \subset V_{J-1} \subset V_J$. A coarse space $V_{\ell-1}$ is obtained from the immediately finer space V_ℓ by replacing all basis functions of level ℓ by their parents. The intergrid transfer operators are deduced from the natural injection $V_{\ell-1} \to V_\ell$ which is readily derived from the algebraic relationships between parent/child basis functions.

The capabilities of the presented numerical scheme are illustrated on various examples, including the numerical simulation of incompressible multiphase flows using the coupling of Navier-Stokes equations with a three-component Cahn-Hilliard model [2]. At the discrete level, a semi-implicit time discretization is used to decouple the two systems within each time step. The resolution of the Navier-Stokes equations is performed by a penalty-projection method [3]. Multigrid preconditioners are used in each inner step: projection of the explicit pressure in the current approximation space, velocity prediction, computation of the pressure increment resolving a Poisson problem and velocity correction. For the resolution of the nonlinear Cahn-Hilliard system, we incorporate multigrid preconditioners in each inner iteration of the Newton linearization method.

Références

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