Discontinuous Galerkin method for two phase flows in heterogeneous porous media

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ABSTRACT

We consider the system of equations describing incompressible immiscible two-phase flow through porous media in global-pressure/fractional flow formulation with capillary pressure, see e.g. [1]. The system consists of an elliptic equation for the pressure and a non-linear parabolic equation for the saturation. The latter equation can degenerate due to capillary pressure effects, so its solution can exhibits shocks. For the numerical solution of this system we develop a sequential scheme with a backward Euler scheme for the saturation equation and a discontinuous Galerkin finite element method for the space approximation of pressure and saturation equations.

Discontinuous Galerkin methods are finite element methods admitting discontinuities at mesh cell interfaces. This allow to use non-matching grids, approximations with varying polynomial order and offers the possibility to implement weakly interface conditions. The substantial flexibility of DG methods makes this approach quite suitable for a large range of computational problems.

We develop a symmetric interior penalty DG method for the global pressure equation and for the diffusion term in the saturation equation, and a DG method with Godunov fluxes for the nonlinear hyperbolic term in the saturation equation. A novel feature is that an accurate total velocity field is recovered from the global pressure equation to be used in the saturation equation [2],[3] that allows avoids any use of limiters in time integration.

In the more complicated case of heterogeneous media we use the interface conditions for pressure and saturation introduced in [4]. The discontinuous Galerkin method techniques are used to weakly implement the prescribed value of the saturation jump at the interface which separates the domains with different capillary pressures. Numerical results showing the advantages of the proposed flux reconstruction and considering oil trapping effects in petroleum reservoir simulation are presented.

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