Benchmark for a coupled diffusion–transport problem

Benchmark group

15 octobre 2012

- Short review of past benchmarks
  - 2D and 3D benchmarks for anisotropic diffusion (FVCA)
  - MOMAS benchmarks
  - SPE benchmarks

- Preliminary results on possible future benchmark
heterogeneous anisotropic diffusion problems on imposed grids
http://www.latp.univ-mrs.fr/latp_numerique

  about 20 schemes

  about 16 schemes

better understanding of relations between schemes, no conclusion on “the best scheme”
loss of simple interpretation for “finite volume methods”
variety of definitions for the “fluxes”

- COUPLEX 1 (simple) and 2 (difficult) : coupling between elliptic equation, and parabolic equation grids to define
- 1D Two-phase gas-water flow benchmarks with dissolution chemistry, mechanics ...

improvements in the choice of primary variables and in the understanding of models with 1D benchmarks
COUPLEX 1 : comparison of engineering techniques as well as numerical schemes, no numerical convergence exhibited
small meshes (except medium size mesh in 1995), no numerical convergence expected
focus on fluid properties modeling and on physics except 1993 and 2001

<table>
<thead>
<tr>
<th>Year</th>
<th>Title</th>
<th>Mesh</th>
<th>Phases</th>
<th>Components</th>
<th>Participants</th>
</tr>
</thead>
<tbody>
<tr>
<td>1981</td>
<td>3D Black-Oil</td>
<td>$10 \times 10 \times 3$</td>
<td>3</td>
<td>3</td>
<td>7</td>
</tr>
<tr>
<td>1986</td>
<td>3Φ coning study</td>
<td>$10 \times 15$ r-z</td>
<td>3</td>
<td>3</td>
<td>11</td>
</tr>
<tr>
<td>1987a</td>
<td>compositional</td>
<td>$9 \times 9 \times 4$</td>
<td>3</td>
<td>5 to 13</td>
<td>9</td>
</tr>
<tr>
<td>1987b</td>
<td>steam injection</td>
<td>$9 \times 5 \times 4$</td>
<td>3</td>
<td>3 + en.</td>
<td>6</td>
</tr>
<tr>
<td>1987c</td>
<td>Miscible Flood</td>
<td>?</td>
<td>2</td>
<td>4?</td>
<td>?</td>
</tr>
<tr>
<td>1990</td>
<td>dual porosity</td>
<td>$10 \times 1 \times 5 \times 2$</td>
<td>3</td>
<td>3</td>
<td>9</td>
</tr>
<tr>
<td>1991</td>
<td>Horizontal wells</td>
<td>$9 \times 9 \times 6$</td>
<td>3</td>
<td>3</td>
<td>14</td>
</tr>
<tr>
<td>1993</td>
<td>Gridding technique</td>
<td>$10 \times 10 \times 4 + X \times 4$</td>
<td>2</td>
<td>2</td>
<td>5</td>
</tr>
<tr>
<td>1995</td>
<td>3D Black-Oil</td>
<td>$24 \times 25 \times 15$</td>
<td>3</td>
<td>3</td>
<td>9</td>
</tr>
<tr>
<td>2001</td>
<td>Upscaling Technique</td>
<td>$100 \times 1 \times 20 + 5 \times 1 \times 5$</td>
<td>3</td>
<td>3</td>
<td>9</td>
</tr>
</tbody>
</table>
Goal: numerical convergence on coupled problem elliptic/(hyperbolic-parabolic)

First step: 2D heterogeneous anisotropic

\[ -\text{div}(\mathbf{F}) = S \text{ in } \Omega \text{ with } \mathbf{F} = -\Lambda(x)m(c)\nabla p \]

\[ \partial_t A(x, c) + \text{div}(c\mathbf{F} - D(x, \mathbf{F})\nabla c) = s - R\ A(x, c) \]

\[ A(x, c) = \phi(x)c + \psi(x)c^\gamma(x) \]

\[ c = \bar{c} \text{ on } \partial\Omega^+ \]

- \( \Omega \subset \mathbb{R}^d \), with \( d = 2 \text{ or } 3 \)
- the function \( m \in C^1(\mathbb{R}) \) is valued in \([m, \bar{m}]\) with \( 0 < m \leq \bar{m} \) (it may be taken constant in some test cases)
- \( \Lambda(x) \) sdp op. \( \mathbb{R}^d \times \mathbb{R}^d \) with \( 0 \leq \lambda|\xi|^2 \leq \Lambda(x)\xi \cdot \xi \leq \bar{\lambda}|\xi|^2 \)
- “diffusion-dispersion” tensor
  \[ D(x, \xi) = d_0(x)\text{Id} + |\xi| \left( \alpha_t(x)\text{Id} + (\alpha_\ell(x) - \alpha_t(x))\frac{\xi\xi^t}{|\xi|^2} \right) \]
- \( \gamma \geq 0 \) is Freundlich parameter \( R > 0 \) half-life
First test case: data from Test 7 of 2D Benchmark

\[-\text{div}(\mathbf{F}) = S \text{ in } \Omega \text{ with } \mathbf{F} = -\Lambda(x)\nabla p\]

\[\partial_t c + \text{div}(c\mathbf{F}) = 0\]

Injection by the left, analytical pressure with parallel isolines

Mesh 1 and pressure field on Mesh 2
Results with SUSHI

Cumuls in $\Omega_i$ and cumulated flux at $\Gamma_4$
Second test case

\[-\text{div}(\mathbf{F}) = S \text{ in } \Omega \text{ with } \mathbf{F} = -\Lambda(x)\nabla p\]

\[\partial_t c + \text{div}(c\mathbf{F}) = 0\]

Injection by the bottom, analytical pressure with combination of parallel and transversal fluxes

Stream lines, pressure field and final concentration field on Mesh2
Case 2 – Cumulated concentration at time $t$ for domain $\Omega_i$, $i = 1, 2, 3$

Case 2 – cumulated Flux through $\Gamma_i$ at time $t$ (logarithm scale for $\Gamma_4$)
Third test case

\[-\text{div}(\mathbf{F}) = S \text{ in } \Omega \text{ with } \mathbf{F} = -\Lambda(\mathbf{x})\nabla p\]

\[\partial_t c + \text{div}(c\mathbf{F} - D(\mathbf{F})\nabla c) = 0\]

\[D(\xi) = d_0\text{Id} + |\xi| \left( \alpha_t\text{Id} + (\alpha_\ell - \alpha_t)\frac{\xi\xi^t}{|\xi|^2} \right)\]

\[c = \bar{c} \text{ on } \partial\Omega^+\]

Similar to case 2: injection by the bottom, analytical pressure with combination of parallel and transversal fluxes

Current lines, pressure field on Mesh2

By diffusion, \(c > 0\) in \(\Omega_3\) (imposed boundary value continuous on \(\partial\Omega\))
Case 3 – Cumulated concentration at time $t$ for domain $\Omega_i, i = 1, 2, 3$

Case 3 – cumulated Flux through $\Gamma_i$ at time $t$ (logarithm scale for $\Gamma_4$)
Improvement of 2D test cases, with more nonlinear features (Roland Masson)
Comeback to COUPLEX1?
pass to 3D, basin geometry

Discussion