ON TAIL PROPERTIES OF STOCHASTIC RECURSIONS CONNECTED WITH GENERALIZED RIGID MOTIONS

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We consider the d-dimensional Euclidean space $V = \mathbb{R}^d$, with the norm:

$$|x| = \Big(\sum_1^d |x_i|^2\Big)^{\frac{1}{2}}.$$

Let H be the group of affine transformations of V defined by

$$hx = gx + q, \quad x \in V$$

where $q \in V$ and $g \in \mathrm{Gl}(V)$ satisfies

$$|gx| = |g||x|, \quad x \in V. \tag{0.1}$$

Then $G = \mathbb{R}_+^* \times O(V)$, where $\mathbb{R}_+^* = \{\lambda I : \lambda > 0\}$. Let $(Q_n, M_n) \in H$, $Q_n \in V$, $M_n \in G$, $n \geq 0$ be a sequence of i.i.d random variables with law μ. We consider the stochastic recursion

$$X_{n+1} = M_{n+1}X_n + Q_{n+1}, \quad n \geq 0, \quad X_0 = Q_0, \tag{0.2}$$

If $\mathbb{E}\log|M_n| < 0$ and $\mathbb{E}\log^+|Q_n| < \infty$ then 0.2 has a unique (in law) stationary solution Z

$$Z = MZ + Q, \text{ (in law)}.$$

Let ν be the law of Z and $\bar{\mu}$ the image of μ under $(v, g) \mapsto g$. Let G_μ be the closed group generated by the support of $\bar{\mu}$ and let $C_\mu = G_\mu \cap O(V)$. The Mellin transform of μ_r is

$$\kappa(s) = \mathbb{E}|M|^s, \text{ for } 0 \leq s < s_\infty,$$

where $s_\infty = \mathrm{supp}\{s \in \mathbb{R}_+ : \kappa(s) < \infty\}$.

Theorem 0.3. *Assume that*

$$\mathbb{E}\log|M| < 0; \tag{0.4}$$

$$\text{for some } \chi > 0,\ \mathbb{E}|M|^\chi = 1; \tag{0.5}$$

$$\mathbb{E}|M|^\chi|\log|M|| < \infty; \tag{0.6}$$

$$\mathbb{E}|Q|^\chi < \infty; \tag{0.7}$$

Assume that $G_\mu = \mathbb{R}_+^* \times C_\mu$ (simplification). Then there is a Radon measure Λ on $V \setminus \{0\}$ such that for $f \in C_c(V \setminus \{0\})$

$$\lim_{|g|\to 0, g\in G_\mu} |g|^{-\chi}\mathbb{E}f(g(Z)) = \lim_{|g|\to 0, g\in G_\mu} |g|^{-\chi} g\nu(f) = \int_V f(x)\, d\Lambda(x), \tag{0.8}$$

Moreover, there is a measure σ_μ on the unit sphere $S^1 = \{x \in V : |x| = 1\}$ such that in polar coordinates

$$\int_V f(x)\, d\Lambda(x) = \int_{S^1\times\mathbb{R}^+} f(au)\, d\sigma_\mu(u)\frac{da}{a^{1+\chi}} \tag{0.9}$$

and, σ_μ is C_μ invariant. If

$$s_\infty = \infty \quad and \quad \sup_{s\geq 0}\left(\frac{\mathbb{E}|Q|^s}{\kappa(s)}\right)^{\frac{1}{s}} < \infty \tag{0.10}$$

then

$$\sigma_\mu = 0 \quad iff \quad \nu = \delta_{x_0}.$$

If G_μ is of the form

$$G_\mu = \Gamma \times C_\mu,$$

$\Gamma = \{\exp tD : t \in \mathbb{R}, D \in Gl(V),$ *semisimple*$\}$ *then*

$$\lim_{|g|\to 0, g\in G_\mu} |g|^{-\chi}\mathbb{E}f(g(Z)) = \lim_{|g|\to 0, g\in G_\mu} |g|^{-\chi} g\nu(f) = \int_V f(x)\ d\Lambda(x), \tag{0.11}$$

and

$$\delta_g * \Lambda = |g|^\chi \Lambda.$$

If 0.10 then

$$\Lambda = 0 \ iff \ \nu = \delta_{x_0}$$

Moreover, 0.10 implies

$$supp\sigma_\mu = \mathbf{pr}\ (S_\infty \cap \overline{supp\nu}) \tag{0.12}$$

To explain 0.12 we denote by S_∞ the shere at infinity of V and we write $\bar{V} = V \cup S_\infty$. The space $\bar{V}$ is endowed with conical topology and we write $\bar{Y}$ for the closure of $Y \subset V$ in $\bar{V}$. We observe that the radial projection **pr** of $\bar{V}$ on S_1, gives a natural identification of S_∞ with $S_1 = \{x \in V : |x| = 1\}$. We write $V \setminus \{0\}$ as the product $V \setminus \{0\} = S_1 \times \mathbb{R}^*_+$ using polar coordinates.

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