Wolfgang WOESS (TU Graz = Graz University of Technology)
"Horocyclic products of trees"

joint work with Laurent BARTHOLDI (Lausanne) and Markus NEUHAUSER (Vienna)

Let $T_1, \dots, T_d$ be homogeneous trees with degrees $q_1+1, \dots, q_d+1 \geq 3$, respectively. For each tree, let $\mathfrak{h} : T_j \to \mathbb{Z}$ be the Busemann function with respect to a fixed boundary point (end). Its level sets are the horocycles. The horocyclic product of $T_1, \dots, T_d$ is the graph $\mathsf{DL}(q_1, \dots, q_d)$ consisting of all d-tuples $x_1 \cdots x_d \in T_1 \times \cdots \times T_d$ with $\mathfrak{h}(x_1) + \cdots + \mathfrak{h}(x_d) = 0$, equipped with a natural neighbourhood relation. We explore the geometric, algebraic, analytic and probabilistic properties of these graphs and their isometry groups. If $d = 2$ and $q_1 = q_2 = q$ then we obtain a Cayley graph of the lamplighter group (wreath product) $(\mathfrak{Z}_q) \wr \mathbb{Z}$. If $d = 3$ and $q_1 = q_2 = q_3 = q$ then DL is the Cayley graph of a finitely presented group into which the lamplighter group embeds naturally. Also when $d \geq 4$ and $q_1 = \cdots = q_d = q$ is such that each prime power in the decomposition of q is larger than $d - 1$, we show that DL is a Cayley graph of a finitely presented group. On the other hand, when the q_j do not all coincide, $\mathsf{DL}(q_1, \dots, q_d)$ is a vertex-transitive graph, but is not the Cayley graph of a finitely generated group; its isometry group is non-unimodular. The ℓ^2-spectrum of the "simple random walk" operator on DL is always pure point. When $d = 2$, it is known explicitly from previous work, while for $d = 3$ we compute it explicitly. Finally, we determine the Poisson boundary of a large class of group-invariant random walks on DL.